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Some Problems Associated with Digital Control of Dynamical Systems

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Digital control of a continuous-time system implies discretization in time of the system. The discretization is likely to bring about changes in the dynamic characteristics of the continuous-time system. Two approaches to the design of a digital control system can be considered: 1) discretization of a previously designed continuous-time control system and 2) direct design in the discrete-time domain. The difference between these two approaches and their effects on a closed-loop control system employing a Luenberger-type observer is demonstrated. The first approach consists of simulating a continuous-time control system on a digital computer. Because the possibility exists that a given set of parameters of the continuous-time system, such as gains and observer poles, will produce a change in the dynamic characteristics of the system during digital simulation, the question is how large the sampling time can be without affecting adversely the discrete simulation. An example of an unstable digital simulation of a closed-loop control system employing a deterministic observer that is stable in continuous-time is presented. The second approach inherently guards against changes in the dynamic characteristics that may be caused by discretization. The two approaches are compared by using a second-order dynamic system.

Introduction

ALMOST all the schemes for the control of large flexible spacecraft that have been reported in the aerospace literature to date are in terms of continuous-time systems of equations, although digital schemes have begun to appear.[1-3] The digitization of a continuous-time dynamical system is necessary if the response is to be simulated on a digital computer. Digitization is also necessary for the design of a sampled-data control system. Whereas the resulting systems in both cases are digital, they may be different dynamically because of the different techniques used to digitize the systems. One way of simulating a continuous-time control system design on a digital computer is to discretize the system by inserting sample and hold steps into the inputs and outputs of the system. We shall refer to the resulting digital system as the discretized version of the continuous-time dynamics. In this paper, we shall concentrate on the discretized version of a continuous-time system. In particular, we shall be concerned with the problems associated with the digitization of a continuous-time control system which uses a deterministic state observer to implement the linear feedback control law.

The most common way of designing sampled-data control systems in modern control theory is to design directly in discrete-time. In designing a digital control loop, a natural question arises whether the discretized version of a previously designed continuous-time control system can be used as a sampled-data control system. References 1 and 4 address a related problem by introducing a digital redesign concept in which a digital system is designed such that the response of the digital system matches the response of a previously designed continuous-time control system at sampling times. It is shown that, except under special conditions, complete matching of the states is not possible, although partial matching of states is possible. The requirement of matching the states of the digital system and the continuous-time system is imposed by the designer and may indeed be so restrictive as to prevent a solution to the problem, as shown in Refs. 1 and 4. Hence, the answer to the question posed above still remains, particularly if one does not insist on complete matching of the states of the continuous-time and the digital system at sampling times. Indeed, a digital system may be able to provide a satisfactory approximation of a continuous-time system. Because the digital system obtained by discretizing the continuous-time equations is only an approximation to the continuous-time system the question as to whether the approximation is satisfactory is vital to the simulation of a continuous-time system on a digital computer. Clearly, this question does not arise in the direct discrete-time domain design of digital control systems, which explains why the direct discrete-time design is preferred almost invariably. Even though the difference between the digital system representing a discretization of a continuous-time design and the direct discrete-time design is obvious, concrete illustrations of the reasons why a direct discrete-time domain design is favored over the discretized version of continuous-time system are absent from the literature. As a result, the question of the interchangeability of the two approaches to the digitization technique is left virtually unanswered quantitatively. In this paper, the effects of digitization on the state estimation error for a control system cascaded with a deterministic state observer are discussed. The two digitization techniques are compared quantitatively and the conditions under which the two methods yield similar results are explored. The simple answer to the question may be given intuitively by considering an arbitrarily small sampling time. However, such an answer is not satisfactory if the sampling time cannot be made arbitrarily small, which is often the case. In the first place, an arbitrarily small sampling time is undesirable if the simulation requires very large overall computational time, as well as large time per computational cycle. The latter is particularly true for real-time control. To reduce computational time, the sampling time must be made reasonably large. But this may conflict with the requirement of accurate digital simulation of the continuous-time system. Hence, the question reduces to how large the sampling time can be made without degrading the simulation.

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It is well known that application of sampling and hold to a continuous-time control system can produce adverse effects on the performance of the system if the dynamics of the system is thereby changed substantially. References 4-6 demonstrate unstable digital systems due to the sampling process in conjunction with high feedback gains. References 4, 7, and 8 discuss the error involved in approximating continuous-time systems by digital systems for single-input, single-output systems. The results are presented by using a transfer function analysis for a feedback control system. Moreover, it should be pointed out that, except for Ref. 2 none of the above references include an observer in the control loop.

In this paper, we present concrete answers to the questions posed above regarding the accuracy of the digital simulations of continuous-time systems and the interchangeability of the digitization techniques for a closed-loop control system with multi-inputs and multi-outputs. The derivations presented in this paper are in the time domain and not in the frequency domain, in contrast to those presented in Refs. 1 and 4-8. The comparison between the continuous-time and digital control systems is made on the basis of the state estimation error. Although the estimation error discussed in this paper contains the effects of digitization implicitly, the error is different in nature than that implied in Refs. 4-8, where in the latter the error represents merely the difference between the continuous-time state (or the output) and the discrete-time state (or the output) of the control system. The derivations presented in this paper demonstrate quantitatively expected ill effects of the sampling process and of estimation errors. This investigation finds its origin in the digital simulation of control systems associated with flexible spacecraft.[1-3]

Discretization of Continuous-Time Control Systems

Let us consider feedback control of a dynamic system in the continuous-time domain. The state equation describing the continuous-time system is

$$\dot{x}=Ax+BX \tag{1}$$

and the measurements vector is

$$z=Cx \tag{2}$$

The system is assumed to be both controllable and observable. The feedback control law is assumed in the form

$$X=K\hat{x}+r(t) \tag{3}$$

where K is the gain matrix and $r(t)$ is a reference input vector. Moreover, $\hat{x}$ is the estimated state vector, as obtained from the observer

$$\dot{\hat{x}}=A_{OC}\hat{x}+B_{OC}z+N_{OC}X \tag{4}$$

in which

$$(A-B_{OC}C)=A_{OC} \qquad N_{OC}=B \tag{5}$$

The subscript O denotes quantities pertaining to the observer and the subscript C denotes continuous-time quantities.

The error vector $\epsilon_C(t)$ between the actual state vector x and the estimated state vector $\hat{x}$ is governed by

$$\dot{\epsilon}_C(t)=\dot{x}-\dot{\hat{x}}=A_{OC}(x-\hat{x})=A_{OC}\epsilon_C(t) \tag{6}$$

If the observer matrix A_{OC} eigenvalues all have negative real parts, then the error vector ϵ_C will approach zero as $t\rightarrow\infty$. Let us consider a given system dynamics and assume that a continuous-time observer has been designed and a feedback control law has been selected. Then, the next task is to simulate the closed-loop control system response. We shall consider a simulation on a digital computer, to be implemented as a sampled-data control system.

The discretized form of Eqs. (1), (4), (2) and (3) can be written respectively

$$x_{k+1}=\Phi x_k+\Gamma X_k \qquad (k=0,1,\ldots) \tag{7}$$

$$\hat{x}_{k+1}=\Phi_{OC}\hat{x}_k+\Gamma_{OC}B_{OC}z_k+\Gamma_{OC}N_{OC}X_k \qquad (k=0,1,\ldots) \tag{8}$$

$$z_k=Cx_k \qquad (k=0,1,\ldots) \tag{9}$$

$$X_k=K\hat{x}_k+r_k \qquad (k=0,1,\ldots) \tag{10}$$

where

$$\Phi=e^{AT} \qquad \Gamma=\int_0^T e^{A\lambda}B\mathrm{d}\lambda \tag{11a}$$

$$\Phi_{OC}=e^{A_{OC}T} \qquad \Gamma_{OC}=\int_0^T e^{A_{OC}\lambda}\mathrm{d}\lambda \tag{11b}$$

in which T is the sampling time. Assuming zero-order hold analog to digital conversion, x_k and X_k are constant over the time interval $kT<t<(k+1)T$. The error between the state vector and its estimate at time $(k+1)T$ is obtained by subtracting Eq. (8) from Eq. (7) and considering Eq. (9), or

$$\begin{aligned}\epsilon_{k+1}&=x_{k+1}-\hat{x}_{k+1}\\&=(\Phi-\Gamma_{OC}B_{OC}C)x_k-\Phi_{OC}\hat{x}_k+(\Gamma-\Gamma_{OC}N_{OC})X_k\end{aligned} \tag{12a}$$

Letting $\epsilon_k=x_k-\hat{x}_k$ be the error at the preceding discrete time, we can replace $\hat{x}_k$ in Eq. (12a) by $x_k-\epsilon_k$. After rearranging, this yields

$$\begin{aligned}\epsilon_{k+1}=&\Phi_{OC}\epsilon_k+[\Phi-(\Phi_{OC}+\Gamma_{OC}B_{OC}C)]x_k\\&+(\Gamma-\Gamma_{OC}N_{OC})X_k\end{aligned} \tag{12b}$$

Equation (12b) implies that the simulation of the estimation error will reduce to zero provided the following two conditions are satisfied: 1) Φ_{OC} has eigenvalues within the unit circle, and 2) the coefficient matrices of the last two terms in Eq. (12b) are zero for nonzero x_k and X_k. Whereas satisfaction of the first requirement presents no difficulty, the second requirement will not necessarily be satisfied, because of the arbitrariness in the choice of the eigenvalues defining the continuous-time observer, and hence of the matrices A_{OC}, B_{OC}, and N_{OC}. Indeed, the coefficient matrices of the last two terms in Eq. (12b) can be rewritten as

$$\begin{aligned}\Delta_1&=\Phi-(\Phi_{OC}+\Gamma_{OC}B_{OC}C)\\&=e^{AT}-e^{A_{OC}T}-\left(\int_0^T e^{A_{OC}\lambda}\mathrm{d}\lambda\right)B_{OC}C\end{aligned} \tag{13a}$$

$$\Delta_2=\Gamma-\Gamma_{OC}N_{OC}=\int_0^T(e^{A\lambda}-e^{A_{OC}\lambda})B\mathrm{d}\lambda \tag{13b}$$

Clearly, there is a degree of arbitrariness in choosing the matrix A_{OC}, where the choice is such that the eigenvalues of the continuous-time observer have negative real parts. It is obvious from Eqs. (13) that the coefficient matrices Δ_1 and Δ_2 do not necessarily vanish. As a result, the last two terms in Eq. (12b) will act as forcing terms, with the implication that the digital simulation will generally contain a certain amount of estimation error. Indeed, if care is not exercised, the error involved in discrete simulation of a continuous-time control can produce responses that are quite different from those of the actual continuous-time system. If the sampling time T is fixed, different choices of observer eigenvalues, and hence of the matrix A_{OC} of the continuous-time system, will yield different coefficient matrices Δ_1 and Δ_2, thus affecting the success of the simulation. This contradicts the original idea that the observer eigenvalues of the continuous-time system can be chosen arbitrarily. At least for discrete simulation purposes this is not so. On the other hand, given a certain set of observer eigenvalues, and hence a certain matrix A_{OC}, changing the sampling time T alone is sufficient to yield different matrices Δ_1 and Δ_2. Mathematically, if the sampling time T approaches zero, the coefficient matrices Δ_1 and Δ_2 will approach zero and the digital estimation error will vanish as $k \to \infty$, so that the discrete system will simulate the continuous system with a large degree of accuracy. But, a reduction in the sampling time T will create problems of a different kind, because an extremely small time increment would increase the computational time for simulation, which also implies a decrease in the available computational time if the computation must be performed within a given amount of time. Furthermore, if the discretized version of the continuous-time equations is to be used as a sampled-data control system, a small sampling time would create problems of implementation. Hence, the problem reduces to that of finding the largest sampling time that is also consistent with successful discrete simulation of a continuous system.

It is perhaps instructive to examine the behavior of the eigenvalues of the closed-loop system of the digital simulation. The control input for the digital simulation is given by

$$X_k = K\hat{x}_k + r_k = K(x_k - \epsilon_k) + r_k \tag{14}$$

Introducing Eq. (14) into Eqs. (7) and (12b) it can be shown that the closed-loop discretized system, subject to the reference input r_k, is

$$\begin{bmatrix} x_{k+1} \\ \hline \epsilon_{k+1} \end{bmatrix} = \left[\begin{array}{c|c} \Phi + \Gamma K & -\Gamma K \\ \hline \Delta_1 + \Delta_2 K & \Phi_{OC} - \Delta_2 K \end{array}\right] \begin{bmatrix} x_k \\ \hline \epsilon_k \end{bmatrix} + \begin{bmatrix} \Gamma \\ \hline \Delta_2 \end{bmatrix} r_k \tag{15}$$

whereas the closed-loop equation for the continuous-time system, subject to the reference input r, can be written in the form

$$\begin{bmatrix} \dot{x} \\ \hline \dot{\epsilon}_C \end{bmatrix} = \left[\begin{array}{c|c} A + BK & -BK \\ \hline 0 & A_{OC} \end{array}\right] \begin{bmatrix} x \\ \hline \epsilon_C \end{bmatrix} + \begin{bmatrix} B \\ \hline 0 \end{bmatrix} r \tag{16}$$

The next step is to determine how the eigenvalues of systems (15) and (16) compare. Equation (16) has its poles in the s-domain, whereas Eq. (15) has its poles in the z-domain. We shall compare the eigenvalues of Eqs. (15) and (16) in the z-domain, which requires the transformation of Eq. (16).[5] Of course, a stable discrete simulation must yield eigenvalues for the closed-loop system within the unit circle and these eigenvalues must compare favorably with the eigenvalues of the continuous-time system, as transformed from the s-domain to the z-domain. In addition, the estimation error vector ϵ_{k+1} of the simulation should remain within an envelope specified by the designer. From Eq. (15), we observe that the vector ϵ_{k+1} is affected by the reference input r_k, whereas from Eq. (16) we conclude that ϵ_C is not affected by r. How well one fulfills these objectives depends on the coefficient matrices Δ_1 and Δ_2, and these matrices depend, in turn, on the sampling time T and the choice of continuous-time observer eigenvalues. If these objectives are fulfilled, and the sampling time T is not too restrictive, then one can implement the discretized equations as a sampled-data system with confidence. It is not possible to render Eq. (12b) free of the last two terms for all sampling times, because this would require that $\Delta_1 = \Delta_2 = 0$, and, from Eq. (13b) this would imply in turn that $A = A_{OC}$, which is of no practical value. The conclusion is that the discrete simulation will always contain an estimation error. Indeed, trying to render the forcing terms in Eq. (12b) equal to zero is equivalent to trying to match the dynamics of the discretized version of the continuous-time control system to dynamics of a control system designed directly in the discrete-time domain. As it turns out, this is possible only for small sampling times.

Direct Design of Discrete-Time Control Systems

Next, let us consider the problem of designing a discrete-time control system directly. To this end, the observer is assumed to have the following dynamics

$$\hat{x}_{D,k+1} = A_{OD}\hat{x}_{D,k} + B_{OD}z_k + N_{OD}X_k \tag{17}$$

where the matrices A_{OD}, B_{OD}, and N_{OD} are determined so as to make the observation error $\epsilon_{D,k+1}$ between the x_{k+1} and $\hat{x}_{D,k+1}$ zero as $k \to \infty$. The subscript D denotes the discrete-time domain quantities. Again the discretized plant dynamics is described by Eqs. (7), (9), and (10). It is assumed that the system of Eqs. (7) and (9) is both controllable and observable. It can be shown that if[6]

$$A_{OD} = \Phi - B_{OD}C \qquad N_{OD} = \Gamma \tag{18}$$

the error $\epsilon_{D,k+1}$ is governed by

$$\epsilon_{D,k+1} = A_{OD}\epsilon_{D,k} \tag{19}$$

The matrix B_{OD} can be determined so as to yield specified eigenvalues for the matrix A_{OD} inside the unit circle in the z-domain. This guarantees that the error tends to zero as $k \to \infty$. This approach obviously circumvents the problems associated with the approach discussed in the previous section. It may prove useful to consider what happens in the two approaches as the sampling time decreases. It is clear that both matrices Δ_1 and Δ_2 tend to zero as the sampling time T approaches zero. Indeed, from Eqs. (13), we obtain

$$\lim_{T \to 0} \Delta_1 = \Phi - (\Phi_{OC} + \Gamma_{OC}B_{OC}C) = 0 \tag{20a}$$

$$\lim_{T \to 0} \Delta_2 = \Gamma - \Gamma_{OC}N_{OC} = 0 \tag{20b}$$

which implies that as $T \to 0$

$$\Phi_{OC} = \Phi - \Gamma_{OC}B_{OC}C \qquad \Gamma_{OC}N_{OC} = \Gamma \tag{21}$$

Hence, using Eqs. (21), as $T \to 0$ Eq. (8) reduces to

$$\hat{x}_{k+1} = (\Phi - \Gamma_{OC}B_{OC}C)\hat{x}_k + \Gamma_{OC}B_{OC}z_k + \Gamma X_k \tag{22}$$

Comparing Eq. (22) with Eq. (17), and considering the forms of matrices A_{OD} and N_{OD}, we deduce that $\Gamma_{OC}B_{OC} \to B_{OD}$, $\Phi_{OC} \to A_{OD}$, so that for small sampling time T the difference between the direct-discrete time design and the discretized-version of the continuous time observer tends to disappear.

Illustrative Example

As an illustration, let us consider the following dynamical system

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} X \tag{23a}$$

$$z = [2 \quad -1]x \qquad X = K\hat{x} \qquad r = 0 \tag{23b}$$

$$x(0) = [5 \;\; 1]^T \tag{23c}$$

Table 1 Effect of sampling time on the dynamic characteristics of the digital simulation ($\gamma_1 = \gamma_2 = 5.0$, $\lambda_1 = \lambda_2 = 5.5$)

T, s	Continuous time poles, transformed to the z-domain	Poles of digital simulation	Δ_1		Δ_2
	0.60653	−0.15769			
0.1	0.60653	0.72000 + j 0.36527	0.15320	0.0	0.07397
	0.57695	0.72000 − j 0.36527	0.21145	0.0	0.10959
	0.57695	0.80806			
	0.77880	0.43680			
0.05	0.77880	0.83465 + j 0.17608	0.04484	0.0	0.02281
	0.75957	0.83465 − j 0.17608	0.06310	0.0	0.03210
	0.75957	0.88726			
	0.95123	0.90749			
0.01	0.95123	0.95692 + j 0.02747	0.00204	0.0	0.00103
	0.94649	0.95692 − j 0.02747	0.00292	0.0	0.00150
	0.94649	0.97020			
	0.99501	0.99272			
0.001	0.99501	0.99620	0.00002	0.0	0.00001
	0.99452	0.99505	0.00003	0.0	0.00001
	0.99452	0.99505			

The feedback control law is implemented by using an observer in the control loop of the type given by Eq. (4), or

$$\dot{\hat{x}} = A_{OC}\hat{x} + B_{OC}z + N_{OC}X \tag{24}$$

where, given the desired pole locations $-\gamma_1$, $-\gamma_2$ for the closed-loop control system and the observer poles $-\lambda_1$, $-\lambda_2$ in the continuous-time domain, in which γ_1, γ_2, λ_2, and λ_2 are positive quantities, the various matrices in Eqs. (22) and (24) can be shown to be

$$K = -[1 + \gamma_1\gamma_2 + (\gamma_1 + \gamma_2) \quad -\gamma_1\gamma_2] \tag{25a}$$

$$B_{OC} = [\tfrac{1}{2}(1 + \lambda_1\lambda_2 + \lambda_1 + \lambda_2) \quad \lambda_1\lambda_2]^T \tag{25b}$$

$$A_{OC} = \begin{bmatrix} -\lambda_1\lambda_2 - (\lambda_1 + \lambda_2) & \tfrac{1}{2}(1 + \lambda_1\lambda_2 + \lambda_1 + \lambda_2) \\ -2\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix} \tag{25c}$$

For the discretized version of the continuous-time design of the control system, if a sampling time T is chosen, then, the matrices Φ, Φ_{OC}, Γ, and Γ_{OC} can be evaluated and the matrices Δ_1 and Δ_2 can be formed. We will first discuss the results by changing the sampling time T only and keeping the continuous-time quantities at specified values. Table 1 displays the results of the simulation. The first two columns present a comparison between the eigenvalues of the digital simulation and the eigenvalues of the continuous-time control loop as transformed to the z-domain. It is observed that both sets of eigenvalues compare quite favorably as the sampling time decreases. This is to be expected, because as $T \to 0$ the discrete simulation approaches the continuous-time behavior. In addition, the matrices Δ_1 and Δ_2, which determine the amount of estimation errors induced by the digital simulation in Eq. (12b), clearly approach zero as $T \to 0$. However, as the sampling time increases, the two sets of eigenvalues differ considerably, and the elements of matrices Δ_1 and Δ_2 increase in value. Hence, for large sampling times, the digital simulation of the continuous-time system is inadequate. Figure 1 shows the simulation results for different sampling times corresponding to the discretized version of the closed-loop controller-observer system, as given by Eq. (15) with $r_k = 0$. For illustration purposes, a nonoscillatory system was chosen, so that the additional constraint of keeping the sampling time to a small fraction of the period of oscillation was obviated. This choice of nonoscillatory response is also consistent with the intention of assessing the observation error resulting from digital simulation alone. For the same reason, the initial error between the estimated state and the actual state was taken to be zero, so that the only estimation error in the control system involved the effect of matrices Δ_1 and Δ_2, and hence it resulted from the digital simulation. The effect of sampling time is depicted in Fig. 1 clearly. It is seen that for a sampling time of $T = 0.01$ s the digital simulation matches the response of the continuous-time system almost exactly. For larger sampling times, the digital simulations show larger amplitudes by as much as 60%. For $T = 0.1$ s the response is oscillatory in nature, which is quite different from the continuous-time response. The acceptability of the degradation of the continuous-time behavior resulting from the digital simulation is, of course, a subjective matter and depends on performance criteria imposed on a specific system. As one case of interest, the discretized version of the continuous-time equations can be used as a sampled-data control system. Plots similar to those of Fig. 1, in this case, will give the designer an idea as to how the digital control system, for the sampling time chosen, is performing in comparison to an ideal response given by the continuous-time response. The choice of very small sampling time may not be desirable, because of hardware requirements. It may not be desirable from a purely computational point of view also, as for large-order highly oscillatory systems, such as a flexible spacecraft, excessive computational times may be necessary. Again, the question reduces to how large the time increments can be made without too much degradation of the actual response.

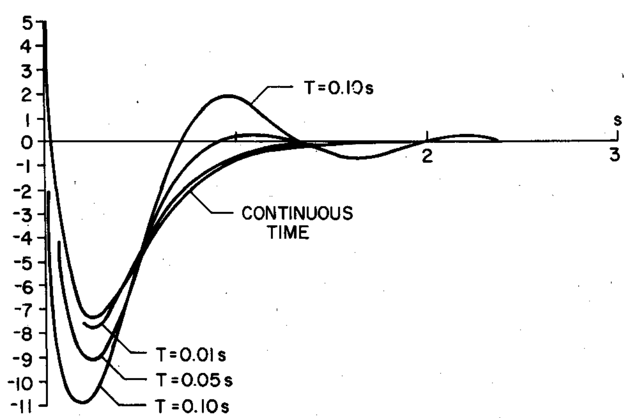

Fig. 1 First component of the state vector for digital simulation of closed-loop continuous-time system employing an observer ($\gamma_1 = \gamma_2 = 5.0$, $\lambda_1 = \lambda_2 = 5.5$).

Table 2 Effect of continuous-time observer eigenvalues on the dynamic characterisics of the closed-loop digital simulation ($T=0.01$ s, $\gamma_1=\gamma_2=5.0$)

$\lambda_1=\lambda_2$	Continuous-time poles, transformed to the z-domain	Poles of digital simulation	Δ_1		Δ_2
	0.95123	−1.86546			
	0.95123	0.84423	0.27061	0.0	0.13584
100.0	0.36788	0.95670 + j 0.01935	0.52848	0.0	0.26528
	0.36788	0.95670 − j 0.01935			
	0.95123	−0.26924			
	0.95123	0.92743 + j 0.05687	0.09419	0.0	0.04726
50.0	0.60653	0.92743 − j 0.05687	0.18040	0.0	0.09953
	0.60653	0.96395			
	0.95123	0.59300			
	0.95123	0.96668	0.01939	0.0	0.00973
20.0	0.81873	0.94329 + j 0.05360	0.03505	0.0	0.01758
	0.81873	0.94329 − j 0.05360			
	0.95123	0.90749			
	0.95123	0.95692 + j 0.02747	0.00204	0.0	0.00103
5.5	0.94649	0.95692 − j 0.02747	0.00292	0.0	0.00150
	0.94649	0.97020			

It was stated in the previous section that the choice of eigenvalues of the continuous-time observer would also affect the success of the digital simulation. This is so because parameters of the observer enter into the computation of the matrices Δ_1 and Δ_2, which in turn affect the eigenvalues of the closed-loop digital simulation, as given by Eq. (15). Of course, as far as the continuous-time design of a deterministic observer is concerned, there are no requirements on the observer poles other than their being faster than the closed-loop poles. Rapid convergence to the actual state requires large eigenvalues. The effect of the observer eigenvalues for a fixed sampling time on the dynamic characteristics of the digital simulation are shown in Table 2 for a sampling time $T=0.01$ s. Note that, previously this sampling time was found to be extremely satisfactory in simulating the response of the continuous-time system (see Fig. 1). An inspection of Table 2 reveals cases which produce eigenvalues for the closed-loop digital system quite different from the transformed poles of the continuous-time system. The difference increases with an increase in the observer eigenvalues. Indeed for $\lambda_1=\lambda_2=100$ (20 times the desired closed-loop eigenvalues) the digital simulation becomes unstable in an oscillatory manner. The elements of matrices Δ_1 and Δ_2 also become larger for large observer eigenvalues, thus increasing the simulated estimation error. Hence, at least in practice, there is no complete arbitrariness in locating the continuous-time observer poles. Table 2 also reveals that, for fixed sampling time, smaller observer eigenvalues render the eigenvalues of the digital simulation closer to those of the continuous system, as transformed to the z-domain. The implication is that larger sampling times can be used for digital simulation without degrading the dynamic characteristics of the continuous-time system provided that smaller eigenvalues for the continuous-time observer are chosen, so that the dynamic characteristics are not changed materially. Therefore, for simulation purposes, smaller observer poles may be more desirable, but this is at the expense of the convergence of the observer to the actual state of the system. Figure 2 shows the estimation error in the digital simulation corresponding to sampling times $T=0.01$ s, 0.1 s and observer eigenvalues −5.5, −5.5. Note that, in the absence of a reference input, the estimation error dies off with time if the system is stable. This is consistent with Eqs. (12a) and (15) (where in the latter $r_k=0$), because for a controlled system the state x_k and the control X_k decrease with time, so that the forcing terms in Eq. (12b) die off as $k\rightarrow\infty$. On the other hand, the presence of a reference input r would excite the estimation error vector of the discretized system continuously, in accordance with Eq. (15), so that the error would not die off. For $T=0.01$ s, corresponding to observer poles −100, −100, the error became unbounded so rapidly that it was not possible to show it in Fig. 2. It should be noted that the choice of continuous-time feedback gain K in Eq. (15) also can affect the dynamics of the digitization adversely. We shall not pursue this here, however.

The alternative method of designing directly in the discrete-time domain avoids these problems, because the estimation error vector approaches zero asymptotically according to Eq. (19). This being the case, the resulting closed-loop system

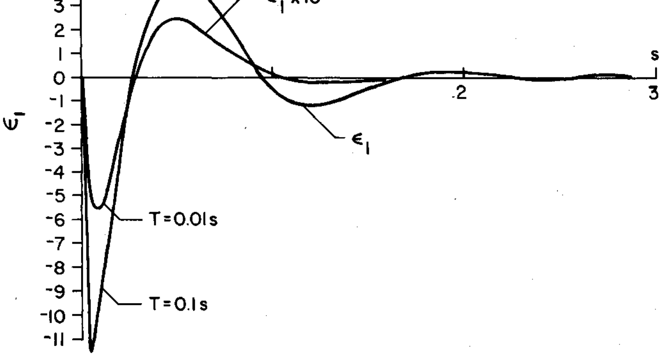

Fig. 2 The error estimation in the first component of the state vector for digital simulation of continuous-time system ($\gamma_1=\gamma_2=5.0$, $\lambda_1=\lambda_2=5.5$).

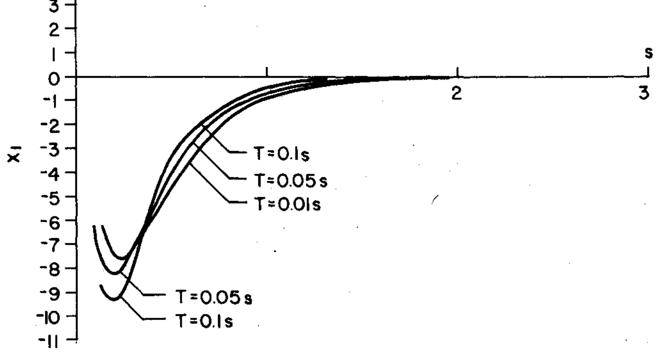

Fig. 3 First component of the state vector for discrete design of observer (plant poles: e^{-5T}, e^{-5T}; observer poles: $e^{-5.5T}$, $e^{-5.5T}$).

Table 3 Comparison of discretized continuous-time observer and direct-discrete observer design ($\gamma_1 = \gamma_2 = 5.0$, $\lambda_1 = \lambda_2 = 5.5$)

T, s	Φ_{OC}		$\Phi - \Gamma_{OC} B_{OC} C$		A_{OD}		$\Gamma_{OC} B_{OC}$	B_{OD}	$\Gamma_{OC} N_{OC}$	$\Gamma = N_{OD}$
0.1	−1.48564	1.21880	−1.33244	1.21880	−1.54786	1.32652	1.21880	1.32652	0.02580	0.10517
	−3.49055	2.63954	−3.27908	2.63954	−3.40353	2.70177	1.63954	1.70177	−0.00959	0.10000
0.01	0.60811	0.19994	0.61016	0.19994	0.60794	0.20105	0.19994	0.20105	0.00902	0.01005
	−0.57262	1.28485	−0.56971	1.28485	−0.57006	1.28503	0.28485	0.28503	0.00854	0.01000

simulation and/or the sampled-data system is expected to be more accurate than that of discretization of a continuous-time design. The observer can be designed directly with poles specified as the z-transforms of the continuous-time poles. Therefore, the difference in the eigenvalues given in Tables 1 and 2 will vanish in a direct discrete-time design, so that the approach inherently guards against alteration of the dynamic characteristics due to changes in the eigenvalues. In particular the unstable digital simulation of the continuous system corresponding to $T = 0.01$ s and $\lambda_1 = \lambda_2 = 100$ can be designed directly in the discrete-time domain without producing instability and hence it can be used successfully to obtain the response by a digital computer. Figure 3 shows the results of simulation for different sampling times. A comparison of Figs. 1 and 3 shows that for all sampling times the direct-discrete design simulation of the continuous-time response is more satisfactory than the simulation of the discretized-continuous time system. We also note that as the sampling time becomes smaller the difference between the direct-discrete design simulation and the discretized continuous-time system simulation tends to vanish, as was ascertained in the previous section. Table 3 shows a comparison of the dynamics of the two approaches for two different sampling times. The matrices of these two approaches resemble each other for smaller sampling time. Hence, we conclude that for small sampling times there is virtually no difference between a direct design in the discrete-time domain and the discretized version of a continuous-time design. However, as the sampling time increases these approaches yield digital systems that are quite different dynamically. Indeed, the same remark can be made in selecting feedback controller gains, both in the discretization of a continuous-time controller and in a direct-discrete design of a controller with discretized continuous-time system, where the controller does not employ an observer in the control loop. References 4-6 illustrate similar instabilities due to the control gains. However, it must be recognized that even in Refs. 4-6 the emphasis is not on comparing the discretized-version of continuous-time equations with a direct-discrete time design.

Conclusion

The digitization of a continuous-time control system employing an observer is considered for digital simulation and design of sampled-data purposes. It is shown that digital simulation of a continuous-time control system cascaded with an observer may exhibit different dynamic characteristics than that of the actual continuous-time system if proper care is not exercised in the choice of sampling time and other control system parameters, such as gains and observer poles. For design of a sampled-data system, either the discretized version of a continuous-time control system or a control system designed directly in the discrete-time domain can be used. However, it is shown that the two approaches can yield significantly different dynamic characteristics, depending on the choice of the sampling time and other design parameters, and the common blithe assumption of simplistic interchangeability between discrete and continuous control design techniques should be given careful scrutiny and perhaps discarded.

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